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ANALYSIS OF THE STRESSED STATE OF A SINGLE-TURN BIMETALLIC SOLENOID
IN AN INTENSE PULSED MAGNETIC FIELD

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One of the characteristic manifestations of interaction of a pulsed electromagnetic field with conductors is Joulean heating which is nonuniform over the conductor thickness. In the design of electrophysical apparatus using large pulsed currents and magnetic fields one must consider the intense heating of the surface of conductive elements which occurs due to the abrupt surface effect. The high heating temperature is a factor which limits capabilities and uses of equipment since it markedly degrades the strength properties of conductive material, which may lead to large deformation and failure of its conductive elements.

Among the components of high power pulsed electrophysical equipment subjected to the most severe loads are single turn solenoids (Fig. 1) intended for repetitive generation of intense magnetic fields ($B_m \leq 50$ T). In such cases heating of the inner surface reaches hundreds of degrees [1]. The mechanical loads produced by electromagnetic field pondermotor forces can be estimated from the maximum magnetic pressure, equal to the magnetic field energy density in the working volume of the solenoid [2]:

$$P_m = B_m^2 / 2\mu_0 \quad (1)$$

(where B_m is the induction amplitude, μ_0 is the magnetic constant of a vacuum). Electrodynamical forces are not the only cause of high mechanical stresses in the solenoid. Upon nonuniform heating of the conductor, produced by the abrupt surface effect, thermoelastic stresses develop, which are determined by the gradient of the temperature distribution over thickness. Since the highest temperature is achieved at the end of the field pulse, when the electromagnetic forces are negligibly small, the latter can be neglected in considering the maximum values of the temperature stresses.

For the abrupt surface effect it is simple to obtain an estimate of the thermoelastic stresses by using Lorentz's expression for a long hollow cylinder nonuniformly heated over wall thickness [3]. The azimuthal and axial stresses on the inner cylinder surface can then be written in the form

$$\sigma_z(R_i) = \sigma_\theta(R_i) = \frac{\beta_0 E}{1-\nu} \left[\frac{2}{R_e^2 - R_i^2} \int_{R_i}^{R_e} \Theta r dr - \Theta(R_i) \right], \quad (2)$$

where R_i , R_e are the inner and outer radii of the cylinder $\Theta = \Theta(r)$ is the temperature distribution over the cylinder wall thickness, β_0 is the coefficient of linear thermal expansion

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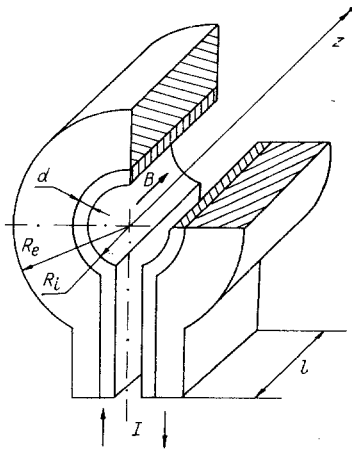


Fig. 1

sion, E is Young's modulus, ν is the Poisson coefficient. To determine the thermoelastic stresses developing in thick-wall ($R_e \gg R_i$) single-turn solenoids in the abrupt surface effect regime, where $\Delta \ll R_i$ (Δ is the electromagnetic field penetration depth into the solenoid wall), we can take $R_e = \infty$ while for the temperature distribution we may take any monotonically decreasing function satisfying the condition $\theta(\infty) = 0$, for example the function

$$\Theta(r) = \Theta_0 \exp(-(r-R_i)/\Delta) \quad (3)$$

(Θ_0 is the temperature on the inner surface of the solenoid $r = R_i$). After substitution of Eq. (3) in Eq. (2) we obtain

$$\sigma_z(R_i) = \sigma_\theta(R_i) = \sigma_0 = -\beta_0 E \Theta_0 / (1 - \nu). \quad (4)$$

To estimate the temperature to which the inner surface is heated by Joulean heating we can neglect the heat transport process, as is permissible in analyzing heating of conductors by an impulsive current [1]. Within the framework of the abrupt surface effect approximation used for the evaluations, Joulean heating of the inner surface can be calculated by solving the equation of penetration of a planar electromagnetic wave into a semi-infinite conductor [2, 4]. The volume heat content on the surface is then

$$Q_0 = \alpha B_m^2 / 2\mu_0. \quad (5)$$

Here the value of the dimensionless coefficient α depends on the form of the pulse and phase of the process. For example, for action on the surface by a unipolar induction pulse of sinusoidal form

$$B_i(t) = \begin{cases} B_m \sin(2\pi t/T), & t < T/2, \\ 0, & t \geq T/2, \end{cases} \quad (6)$$

for $t = T/2$, $\alpha = 2.18$. For the case often met in practice of an exponentially decaying oscillatory pulse

$$B_i(t) = B_0 \sin(2\pi t/T) \exp(-2\delta t/T) \quad (7)$$

(B_0 is the oscillation amplitude, T , δ are the oscillation period and decrement) at $t = \infty$, according to [4],

$$Q_0 = \alpha B_0^2 / 2\mu_0, \quad \alpha = \frac{\pi}{2\delta} \left(\frac{1}{2} - \frac{1}{\pi} \arctg \frac{1 - (\pi/\delta)^2}{2\pi/\delta} \right).$$

Using the relationship $Q_0 = \gamma c_\rho \theta_0$ (γ is density, c_ρ is specific heat) and substituting Eq. (5) in Eq. (4), we find

$$\sigma_0 = \beta_0 E \alpha B_m^2 / [2(1 - \nu) \gamma c_\rho \mu_0]. \quad (8)$$

Values of the relative quantity $\sigma_0' = |\sigma_0 / P_m|$ at $\alpha = 2.18$ (unipolar pulse) for some metals are presented in Table 1.

TABLE 1

| Material | σ_0' | B_* , T |
|-----------------|-------------|-----------|
| Copper | 2,2 | 9 |
| Brass | 2,44 | 16 |
| Hard bronze | 2,12 | 35 |
| Aluminum | 2,86 | 5 |
| Stainless steel | 1,59 | 30 |

Loss of strength and destruction of the metal under repeated loading occurs in the presence of load-unload cycles reaching into the plastic deformation region, and manifests itself by formation and growth of cracks [5, 6]. In the case under consideration such cycles can develop in the solenoid surface layer at $P_m > \sigma_T$ (where σ_T is the yield point), when due to interaction of electromagnetic forces there develops an azimuthal tension exceeding the elastic limit, with subsequent unloading as the induction in the working volume decreases. This cycle is synchronized to the change in magnetic field on the inner surface of the solenoid. Cycles of a second type may occur after completion of the pulse at $|\sigma_0| > \sigma_T$ when plastic compression deformations develop in the heated surface layer, and after equalization of the temperature distribution and cooling, unloading occurs in the elastic region, ending by appearance of residual tensile stresses and their partial or complete relaxation [5, 6]. Cycles of the second type are more dangerous since in this case the plastic deformations change in sign [6]. As follows from Table 1 for the majority of metals thermoelastic stresses are more than twice as large as the maximum magnetic pressure P_m . Therefore temperature stresses are the dominant factor in the appearance of plastic deformations.

Using data on the yield point of metals σ_T and Eqs. (1) and (8), we can calculate the limiting value of induction B_* which will not lead to appearance of plastic deformations of the solenoid: $B_* = \sqrt{2\sigma_T\mu_0(1-\nu)\gamma c\rho/(\beta_0 E\alpha)}$.

For the unipolar pulse of Eq. (6), where $\alpha = 2.18$, values of the limiting induction are presented for several metals in Table 1. These values are somewhat lower than the magnetic strength limit presented in [1], since to calculate the latter failure strengths were used, higher than the corresponding σ_T , with values of magnetic pressure on the conductor surface less than the absolute value of the maximum temperature stress. It is evident from Table 1 that the induction which can be produced repeatedly in solenoids of ordinary conductive materials without plastic deformation comprises some 30 T. As the above analysis shows, increase in the permissible value of B_* can be achieved if we insure a lower temperature on the inner surface of the solenoid.

The severely inhomogeneous current density and heat liberation distributions over conductor thickness produced by eddy currents which lead to a temperature profile of the form of Eq. (3) can, within certain limits, be equalized by using inhomogeneous conductors with resistivity decreasing with depth. In this case the process of current pulse penetration is distinguished by the fact that the current is concentrated near the surface where the resistivity is relatively high for a shorter time, and moves into layers further removed from the surface with better electrical conductivity. Thus a more uniform distribution of current density and heat liberation is achieved, and thus, a reduction in the maximum temperature to which the conductor is heated by the current pulse.

It is evident from analysis of regimes of impulsive electromagnetic field penetration into inhomogeneous conductors with the above properties that in a system with smooth dependence of the resistivity $\rho(x)$ a three-fold reduction in maximum temperature is possible, while in the simplest bimetallic conductor where the resistivity on the surface is greater than in the depths ($\rho_1 > \rho_2$) a reduction in maximum temperature by 30% can be achieved [7-9]. Naturally, the bimetallic conductor, being the simplest system with inhomogeneous electrical conductivity which can be realized in practice, is of the greatest interest for detailed analysis. The goal of such an analysis is acquisition of quantitative characteristics of the stressed state of bimetallic solenoids, as well as determination of the limiting values of magnetic field induction which do not lead to appearance of plastic deformation, i.e., determination of the capabilities of a bimetal in systems for repeated generation of intense magnetic field pulses.

Equalization of temperature distributions and reduction in maximum temperature in the bimetal together with increase in strength characteristics creates conditions for decrease in maximum thermoelastic stresses. However the differences in mechanical characteristics between the components of the bimetallic conductor may lead to the opposite effect, with thermoelastic stresses increasing. In connection with this, to select parameters to optimize the bimetallic solenoid it is necessary to carry out a detailed analysis of its thermally stressed state, caused by mechanical and thermal action of the impulsive electromagnetic field. If we neglect edge effects, which is permissible for relatively long solenoids ($l \geq 3R_i$), we can limit ourselves to a one-dimensional formulation of the problem. Then the mathematical model can be expressed in the form

$$\frac{\partial B}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[\frac{r}{\mu_0} \rho_0(r) (1 + \beta_1(r) Q) \frac{\partial B}{\partial r} \right]; \quad (9)$$

$$\frac{\partial Q}{\partial t} = \frac{\rho_0(r) [1 + \beta_1(r) Q]}{\mu_0^2} \left(\frac{\partial B}{\partial r} \right)^2; \quad (10)$$

$$\gamma \frac{\partial^2 U}{\partial t^2} = \frac{\partial}{\partial r} (\sigma_r - P_m) + \frac{\sigma_r - \sigma_\theta}{r}, \quad P_m = B^2/2\mu_0; \quad (11)$$

$$\sigma_r = \frac{E(r)}{1 - \nu^2(r)} \left[\frac{\partial U}{\partial r} + \nu(r) \frac{U}{r} - (1 + \nu(r)) \beta_0(r) Q \right]; \quad (12)$$

$$\sigma_\theta = \frac{E(r)}{1 - \nu^2(r)} \left(\frac{U}{r} + \nu(r) \frac{\partial U}{\partial r} - (1 + \nu(r)) \beta_0(r) Q \right), \quad (13)$$

where Eq. (9) is the equation of electromagnetic field penetration into the solenoid wall; Eq. (10) is the equation of conductor heating by the pulsed current; Eqs. (11)-(13) are the equations of the dynamic theory of elasticity. The resistivity ρ_0 , the thermal resistivity coefficient β_1 , Young's modulus E , the Poisson coefficient ν , the linear thermal expansion coefficient β_0 , and the density γ depend on r and are characterized by the coating material for $R_i \leq r \leq R_i + d$ and the base material for $r > R_i + d$ (R_i is the inner radius of the solenoid, d is the coating thickness). By solving Eqs. (9)-(13) we find the distribution of the magnetic field induction (axial component) $B(r, t)$, the volume heat content $Q(r, t) = c_0 \gamma \theta$, the temperature $\theta(r, t)$, the displacement $U(r, t)$, the azimuthal and radial mechanical stresses $\sigma_\theta(r, t)$, $\sigma_r(r, t)$. For the solution the system must be completed by corresponding boundary conditions, which for Eq. (9) can be taken in the form

$$B(R_i, t) = B_i(t) \quad (14)$$

[$B_i(t)$ is a given function of time, describing the magnetic field pulse in the solenoid working volume], $B(R_e, t) = 0$ (R_e is the outer radius of the solenoid). For Eq. (11) the boundary conditions can be formulated as the condition on the free boundary $\sigma_r(R_i) = 0$, as well as rigid attachment of the external solenoid surface $U(R_e) = 0$. System (9)-(13) is quite complex and contains many parameters. Therefore it is expedient to use numerical methods for its solution. In the present study to integrate Eqs. (9)-(13) use was made of implicit difference methods with central differences over a spatial variable, providing strong stability of the computation for an arbitrary step in time Δt [10].

In view of the fact that system (9)-(13) admits a large number of variations in the input parameters, describing the mechanical, electrical, and geometrical properties of the object modeled, acquisition of generalized results from the numerical calculations by similarity theory methods is not possible. Therefore it becomes desirable to limit ourselves to detailed examination of a specific example characterizing typical conditions and problems which arise in designing equipment for repeated generation of intense magnetic field pulses. For this example we will use the plasma compression heating solenoid of the TL-1 apparatus, designed for creation of a damped oscillatory magnetic field pulse of the form of Eq. (7) with parameters $B_m = 37$ T, $T = 60$ μ sec, $\delta = 0.7$ [11, 12]. Basic dimensions of the solenoid are as follows: length 180 mm, inner radius 55 mm, thickness of conductive portion of the walls 15 mm. When beryllium bronze (BrBe) is used for the conductive material the maximum inner surface temperature reaches 540°C (Fig. 2).

Figure 2 shows the calculated distributions over solenoid thickness of the temperature (curves 1, 3) and azimuthal stress (curves 2, 4, 5) at a time corresponding to completion of magnetic field pulse action $t = T_p = 7T/2$ for a homogeneous bronze (BrBe) solenoid (lines 1, 2), and a two-layer solenoid (stainless steel-BrBe) with steel coating thickness $d = 1.5$ mm which provides the greatest reduction in maximum local heat liberation (lines 3, 4), and a two-layer solenoid with electrical characteristics of the stainless steel-BrBe pair and

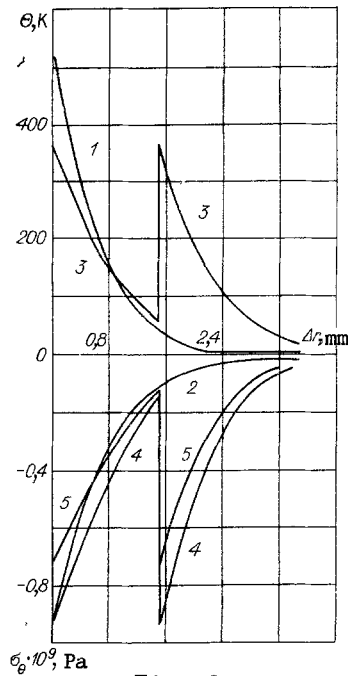


Fig. 2

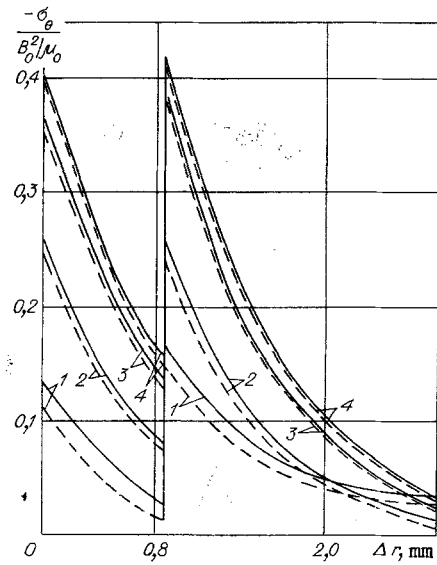


Fig. 3

coating mechanical characteristics equal to those of the base (BrBe) (line 5). Comparison of the curves shown in Fig. 2 permits the conclusion that use of materials for the bimetal components with similar mechanical properties produces a situation where the coating layer thickness optimal in the sense of minimizing the maximum azimuthal stress also insures a minimum maximum temperature.

An example of bimetallic conductor components having similar mechanical properties might be the following pair of metals. For the base we use phosphor bronze with $\rho_0 = 0.11 \cdot 10^{-6} \Omega \cdot m$, $\beta_1 = 0.46 \cdot 10^{-9} m^3/J$, $\gamma = 8.9 \cdot 10^3 kg/m^3$, $E = 0.12 \cdot 10^{12} Pa$, $\nu = 0.38$, $\beta_0 = 0.52 \cdot 10^{-11} m^3/J$, while for the coating having a higher value of resistivity we use German silver (60% Cu, 15% Ni, 25% Zn) with $\rho_0 = 0.45 \cdot 10^{-6} \Omega \cdot m$, $\beta_1 = 0.88 \cdot 10^{-10} m^3/J$, $\gamma = 8.4 \cdot 10^3 kg/m^3$, $E = 0.116 \cdot 10^{12} Pa$, $\nu = 0.34$, $\beta_0 = 0.59 \cdot 10^{-11} m^3/J$.

Analyzing the stressed state of the pulsed solenoid and the change in mechanical stress over the action time by numerical solution of Eqs. (9)-(13), with strict consideration of possible mechanical wave processes, it is desirable to estimate the role of dynamic effects. For this purpose Fig. 3 (lines 1-4 for $t = 13, 30, 60, 210 \mu sec$) shows distributions of azimuthal stress in a bimetallic solenoid (German silver-BrRe) optimized for minimum maximum temperature, as calculated by dynamic equation (11) (dashed curves) and from the equation of the static theory of elasticity (solid lines). Since the time for a sonic wave to travel between the solenoid walls [$t_n = (R_e - R_i)/c_s \approx 5 \mu sec$] is small in comparison with the characteristic time parameters of the magnetic field pulse, calculations by the static theory do not lead to intense differences from the exact calculations. A similar comparison of radial stress distributions (Fig. 4, where lines 1-3 are for $t = 30, 60, 210 \mu sec$) permits the conclusion that for calculation of mechanical stresses and analysis of limiting solenoid parameters it is sufficient to limit oneself to use of the static theory of elasticity, since the most "severe" regime develops at the end of the magnetic field pulse, when solenoid heating is maximal, and mechanical stresses can be considered established.

It is desirable to introduce some criterion to evaluate the limiting solenoid regimes. In accordance with the above, for this criterion we may use the absence of plastic deformations in the solenoid volume, which in Mises-Henky formulation has the form [13]

$$\sqrt{I_2} < \sigma_T \sqrt{2}, \quad (15)$$

where $I_2 = (\sigma_r - \sigma_z)^2 + (\sigma_r - \sigma_\theta)^2 + (\sigma_z - \sigma_\theta)^2$ is the second invariant of the stress tensor. To calculate I_2 in place of σ_θ and σ_r , it is necessary to know the axial stress at an arbitrary point r . To calculate σ_z according to [3], we must use the condition of equality to zero of the total axial force and normal stresses σ_z on the free ends of the solenoid. Then it is easy to obtain [3]

$$\sigma_z = v(\sigma_r + \sigma_\theta) - \beta_0 EQ + \frac{2}{R_e^2 - R_i^2} \int_{R_i}^{R_e} [v(\sigma_r + \sigma_\theta) - \beta_0 EQ] r dr.$$

Thus the condition insuring repeated generation of a given magnetic field pulse is satisfaction of inequality (15) at any moment in time. In practice Eq. (15) will control after completion of the field pulse at $t \geq T_p$, when I_2 has its highest value, as is evident from Fig. 5, which shows the time dependence of the maximum over solenoid thickness of I_2 , calculated for a homogeneous solenoid (BrBe) at $B_0 = 30$ T. The change in slope of the curve in Fig. 5 corresponds to the moment of maximum magnetic field induction on the inner surface of the solenoid.

The mathematical model under consideration allows not only parameter optimization and comparison of variants of single-turn pulsed solenoids, but with use of Eq. (15) it permits using a series of numerical calculations to determine the limiting magnetic field induction for a pulse of specified form. If we then consider that due to reduction in temperature together with mechanical stresses there is an increase in σ_T , then one can expect a more significant effect from using a bimetallic conductor to prepare the single-turn solenoid. As an example we will consider the induction amplitude for a solenoid with the parameters presented above. To analyze condition (15), we will use the dependence of the yield point on specific volume heat content in the form of [14], $\sigma_T = \sigma_{T_0} \exp(-\epsilon Q)$.

Since the literature contains no detailed descriptions of the temperature dependences of yield point for the majority of bronzes and German silver, for the constants in the equation we will use characteristic values for hard bronzes $\sigma_{T_0} \approx 10^9$ Pa, $\epsilon \approx 0.6 \cdot 10^{-9}$ m³/J [1, 15]. In view of this approximation the analysis below is not exhaustively precise, although it will permit quantitative estimates of the efficiency of using a bimetallic conductor. It is to be understood that the pair phosphor bronze-German silver is not the only one in which the temperature reduction effect will be realized. Obviously there are other metals with similar mechanical characteristics and different electrical resistivities, for example, if we consider various types of bronze.

Introducing a quantity calculated at the moment of completion of field pulse action $t = T_p$ in the form $R = \{\sigma_T - \sqrt{I_2/2}\}_{\min}$, $r \in (R_i, R_e)$, we can find the limiting induction pulse amplitude $B_0 = B_{*}$ satisfying Eq. (15), commencing from the equation $R(B_0) = 0$. To do this dependences $R(B_0)$ are constructed in Fig. 6 for a homogeneous bronze solenoid (line 1) and a bimetallic solenoid optimized for minimum maximum value of I_2 with the largest instantaneous induction in the working volume of 37 T (line 2). Comparison of the curves in Fig. 6 shows

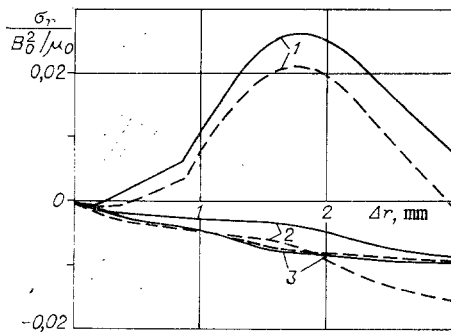


Fig. 4

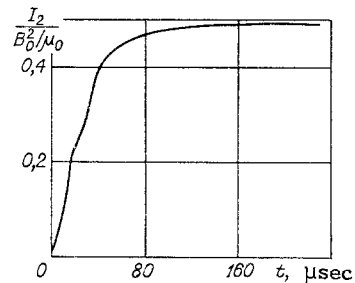


Fig. 5

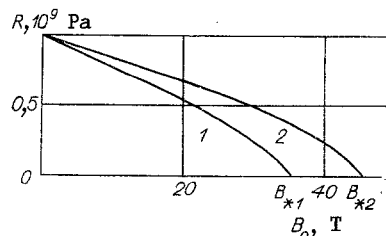


Fig. 6

that the limiting amplitude in the bimetallic solenoid B_{*2} is 1.3 times higher than in the homogeneous solenoid, B_{*1} . It is obvious that the amount of increase in B_* in the bimetallic solenoid depends on the rate of decrease of the yield point with increase in temperature. The minimum effect occurs when σ_T is temperature independent ($\varepsilon = 0$). In this case reduction in mechanical stresses by a factor of 1.3 times in the optimized solenoid corresponds to an increase in permissible B_* by 15%.

The analysis performed above demonstrates the advantages of bimetallic solenoids for use in generation of a large number of intense magnetic field pulses. The greatest effect is achieved in the case where mechanical characteristics, including the yield point of the coating and base materials, are similar to each other. In studying the possibilities of such solenoids as intense magnetic field sources, when generating a limited number of pulses one can allow the appearance of small plastic deformations, consideration of which is an independent problem.

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